Programming Abstractions

Week 3-2: Folds

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Lots of similarities between functions

`(sum lst)`

```
(define (sum lst)
  (cond [(empty? lst) 0]
        [else (+ (first lst)
                 (sum (rest lst)))]))
```

Lots of similarities between functions

`(length lst)`

```
(define (length lst)
  (cond [(empty? lst) 0]
        [else (+ 1
                 (length (rest lst)))]))
```

Lots of similarities between functions

`(map proc lst)`

```
(define (map proc lst)
  (cond [(empty? lst) empty]
        [else (cons (proc (first lst))
                    (map proc (rest lst)))]))
```

Lots of similarities between functions

`(remove* x lst)`

```
(define (remove* x lst)
  (cond [(empty? lst) empty]
        [(equal? x (first lst) (remove* x (rest lst))]
        [else (cons (first lst)
                    (remove * x (rest lst)))]))
```

Let's rewrite this one to look more like the others

```
(define (remove* x lst)
  (cond [(empty? lst) empty]
        [else (if (equal? x (first lst))
                  (remove* x (rest lst))
                  (cons (first lst)
                        (remove* x (rest lst))))]))
```

Some similarities

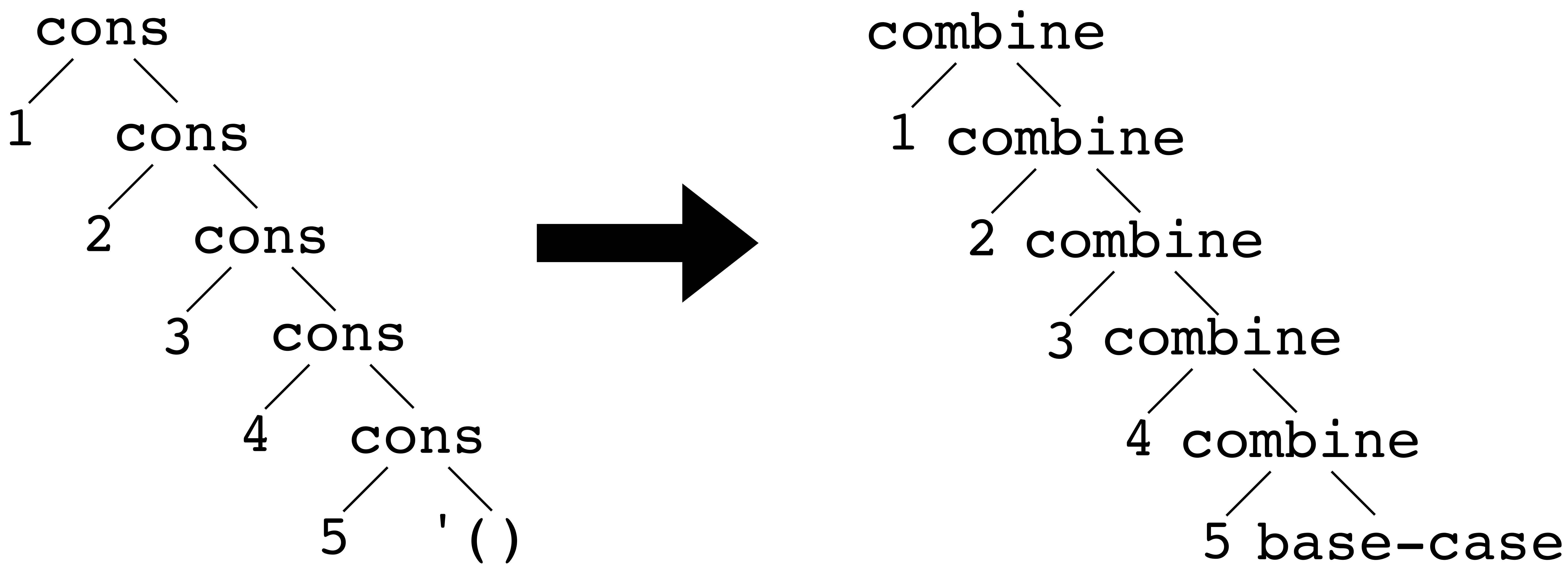
Basic structure is the same (rewriting slightly)

```
(define (fun … lst)
  (cond [(empty? lst) base-case]
        [else
         (let ([head (first lst)]
               [result (fun … (rest lst))])
           (combine head result))]))
```

Function	base-case	(combine head result)
sum	0	(+ head result)
length	0	(+ 1 result)
map	empty	(cons (proc head) result)
remove*	empty	(if (equal? x head) result (cons head result))

Abstraction: fold right

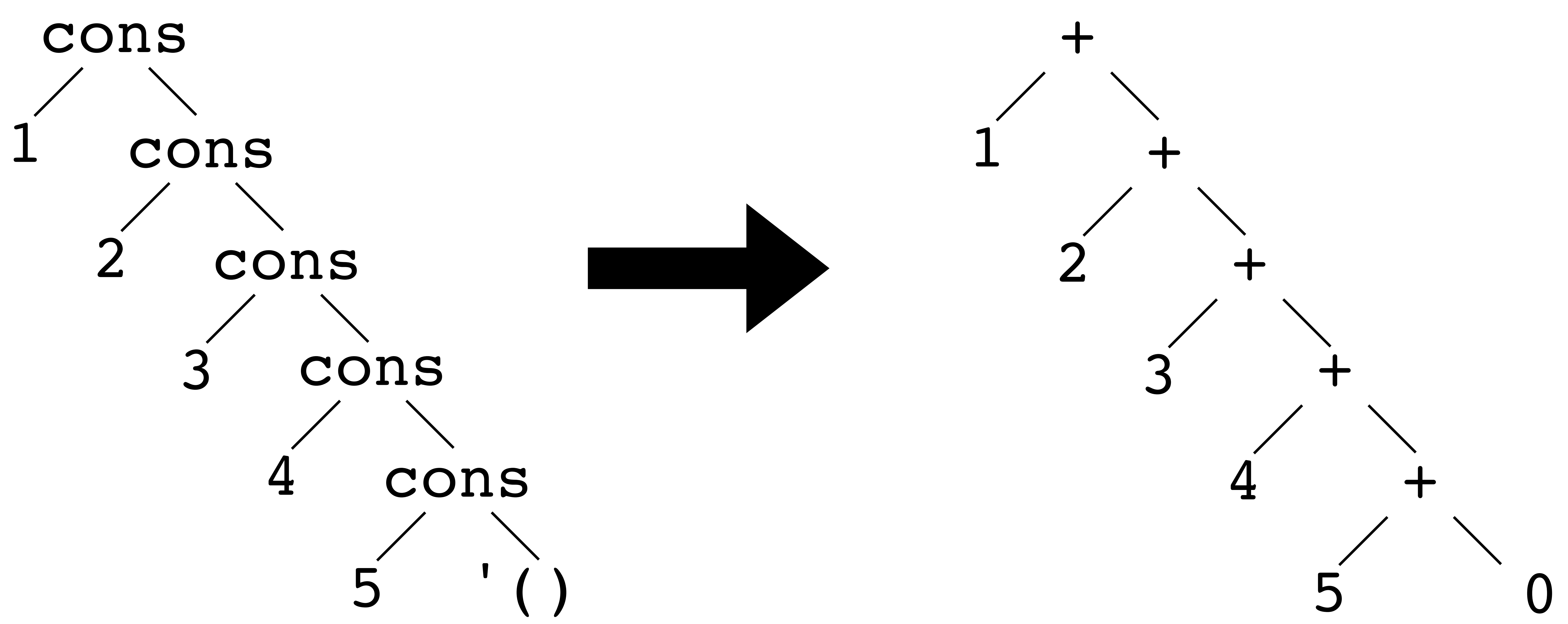
`(foldr combine base-case lst)`



sum as a fold right

`(foldr combine base-case lst)`

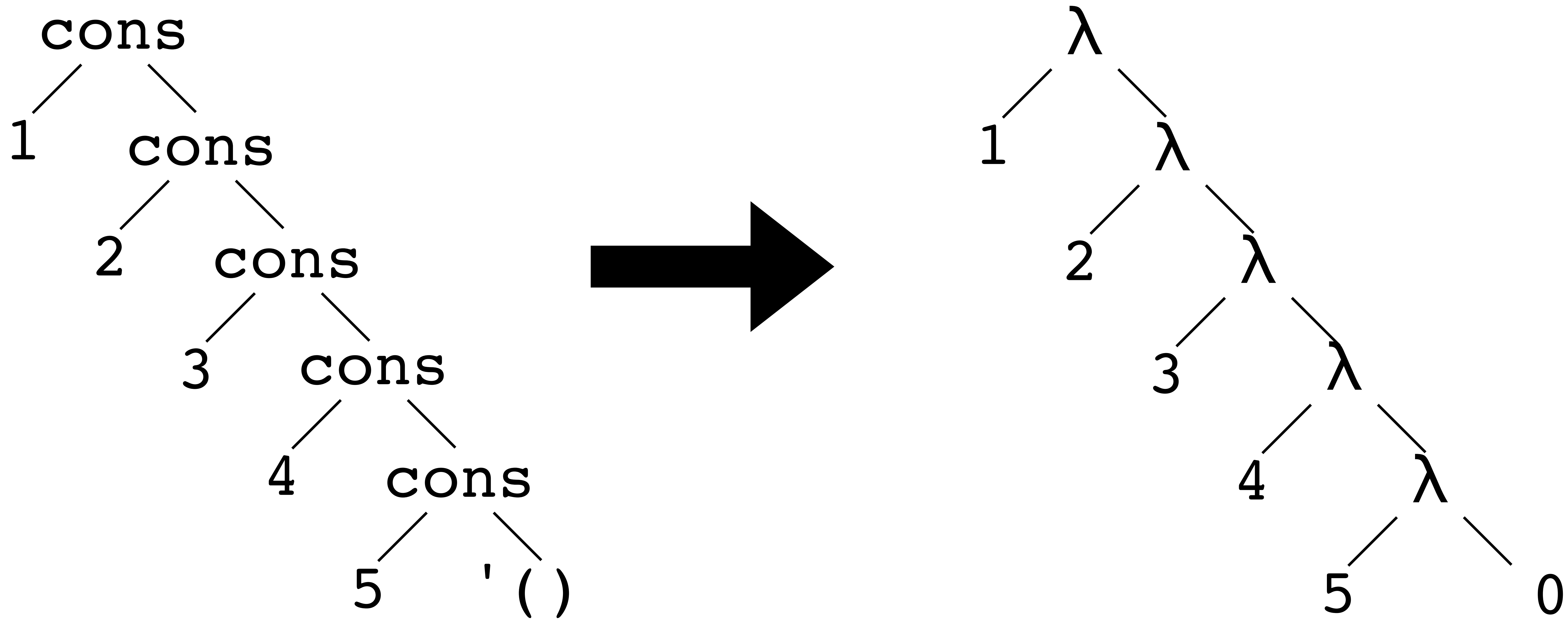
```
(define (sum lst)
  (foldr + 0 lst))
```



length as a fold right

`(foldr combine base-case lst)`

```
(define (length lst)
  (foldr (λ (head result) (+ 1 result)) 0 lst))
```



`map` and `remove*` as fold right

`(foldr combine base-case lst)`

```
(define (map proc lst)
  (foldr (λ (head result)
           (cons (proc head) result))
         empty
         lst))

(define (remove* x lst)
  (foldr (λ (head result)
           (if (equal? x head)
               result
               (cons head result)))
         empty
         lst))
```

Consider the procedure

```
(define (foo lst)
  (foldr (λ (head result)
           (+ (* head head) result))
         0
         lst))
```

What is the result of `(foo '(1 0 2))`?

A. `'(1 0 2)`

B. `'(5 4 4)`

C. `5`

D. `1`

E. None of the above

Consider the procedure

```
(define (bar x lst)
  (foldr (λ (head result)
           (if (equal? head x) #t result))
         #f
         lst))
```

What is the result of `(bar 25 '(1 4 9 16 25 36 49))`?

A. `'(#f #f #f #f #t #f #f)`

B. `'(#f #f #f #f #t #t #t)`

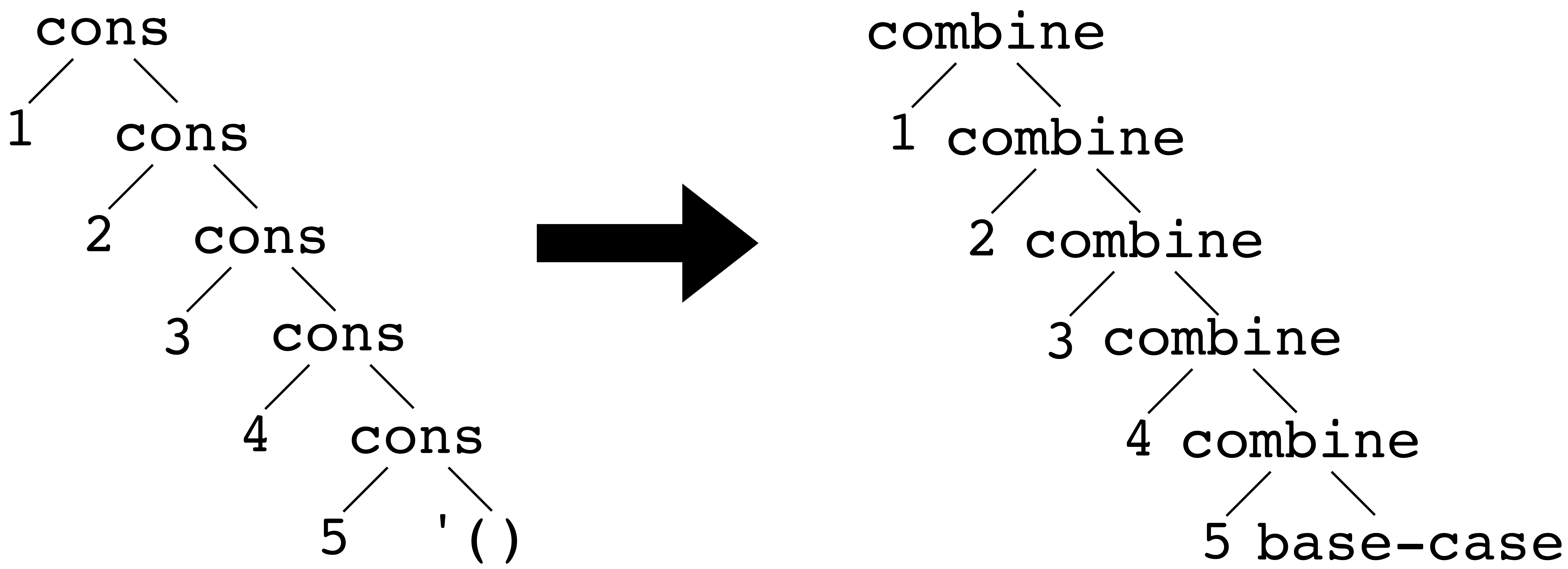
C. `#f`

D. `#t`

E. None of the above

Let's write `foldr`

`(foldr combine base-case lst)`



Accumulation-passing style similarities

```
(define (product lst)
  (define (product-a lst acc)
    (cond [(empty? lst) acc]
          [else (product-a (rest lst)
                           (* (first lst) acc))]))
  (product-a lst 1))
```

Accumulation-passing style similarities

```
(define (reverse lst)
  (define (reverse-a lst acc)
    (cond [(empty? lst) acc]
          [else (reverse-a (rest lst)
                           (cons (first lst) acc))]))
  (reverse-a lst empty))
```

Accumulation-passing style similarities

```
(define (map proc lst)
  (define (map-a lst acc)
    (cond [(empty? lst) acc]
          [else (map-a (rest lst)
                       (cons (proc (first lst)) acc))]))
  (reverse (map-a lst empty)))
```

Some similarities

Basic structure is the same (rewriting slightly)

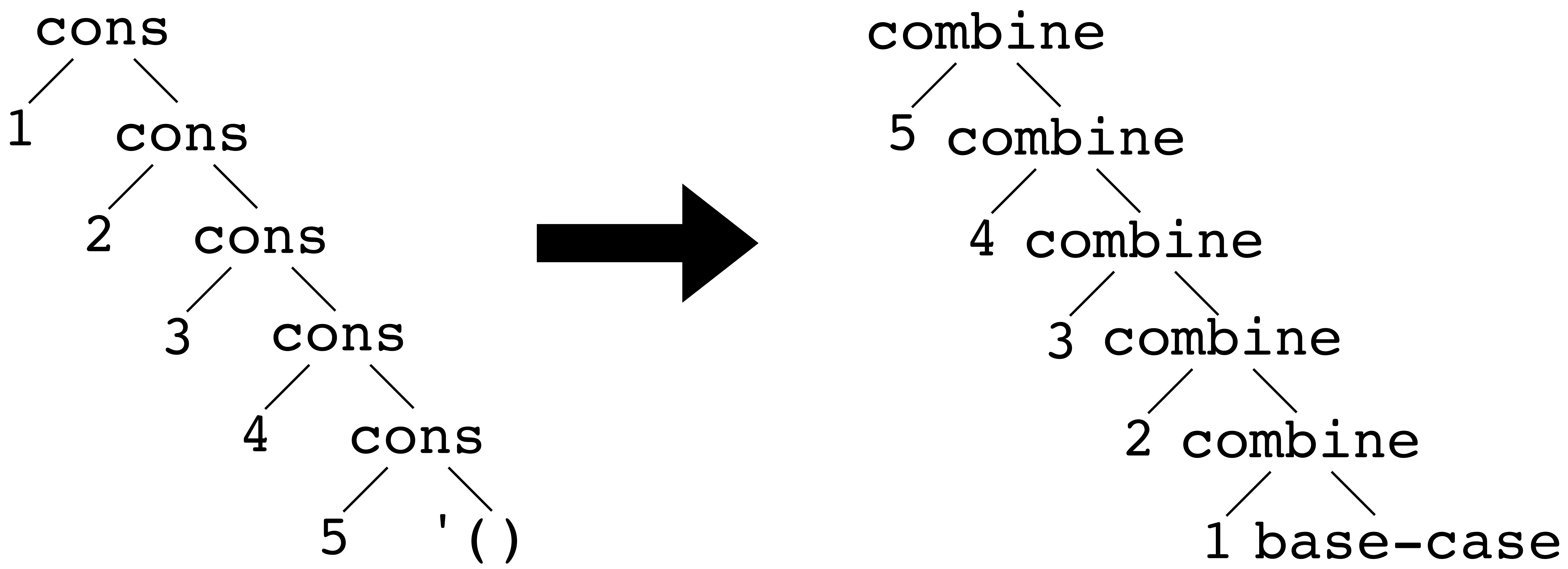
```
(define (fun … lst)
  (define (fun-a lst acc)
    (cond [(empty? lst) acc]
          [else
           (fun-a (rest lst)
                  (combine (first lst) acc))]))
  (fun-a lst base-case))
```

Function	base-case	(combine head acc)
product	1	`(* head acc)`
reverse	empty	`(cons head acc)`
map	empty	`(cons (proc head) acc)`

We must reverse the result

Abstraction `foldl`

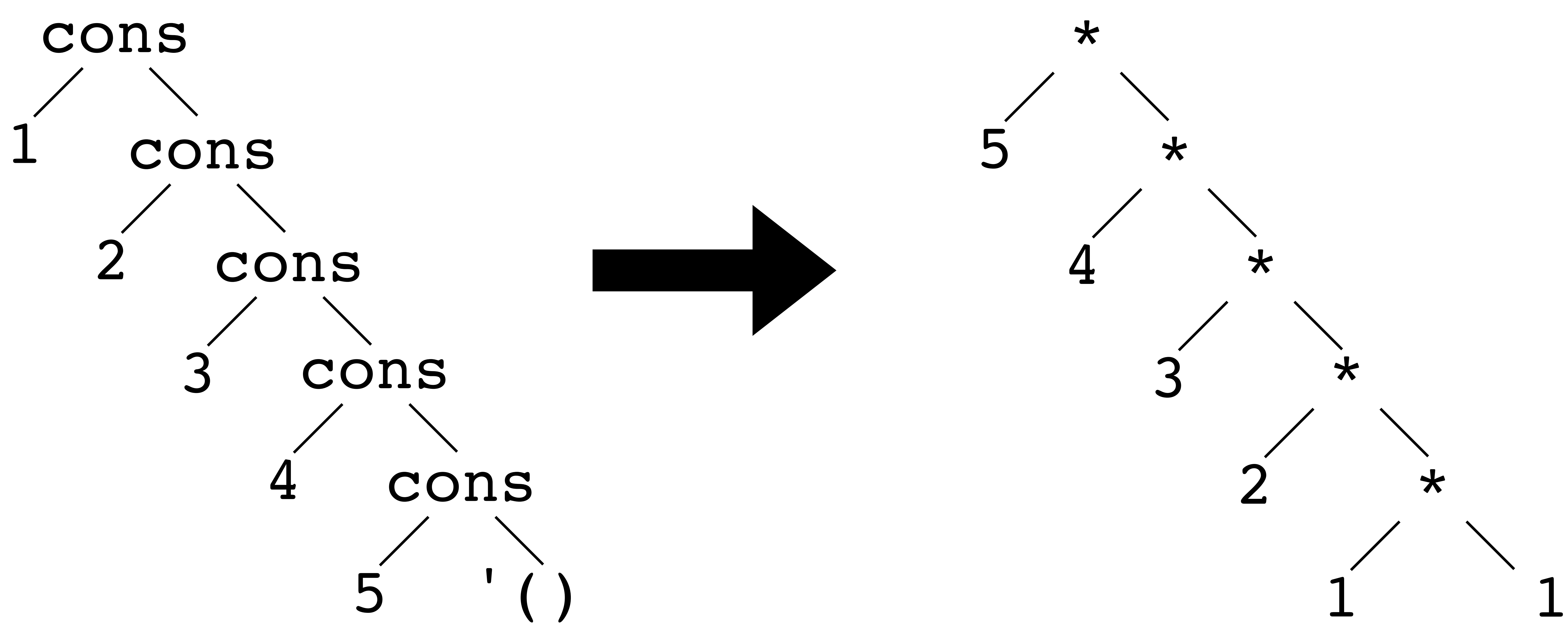
`(foldl combine base-case lst)`



product as fold left

`(foldl combine base-case lst)`

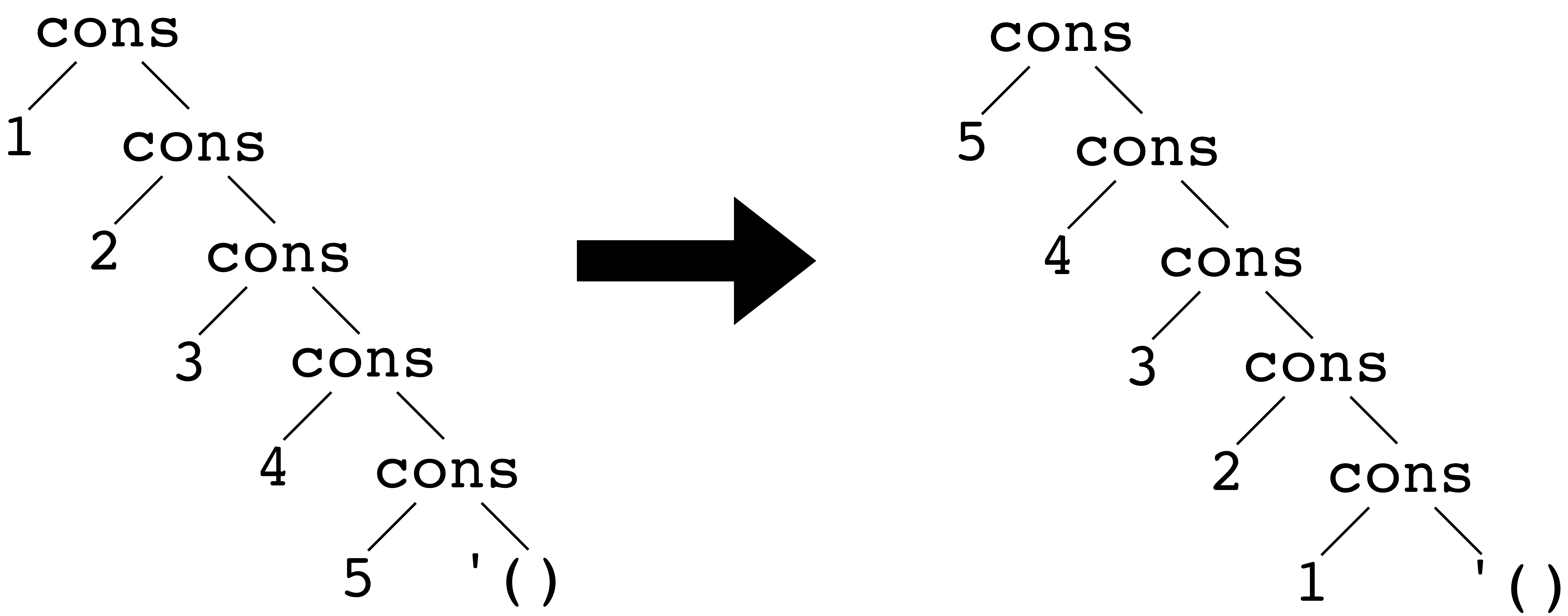
```
(define (product lst)
  (foldl * 1 lst))
```



reverse as fold left

`(foldl combine base-case lst)`

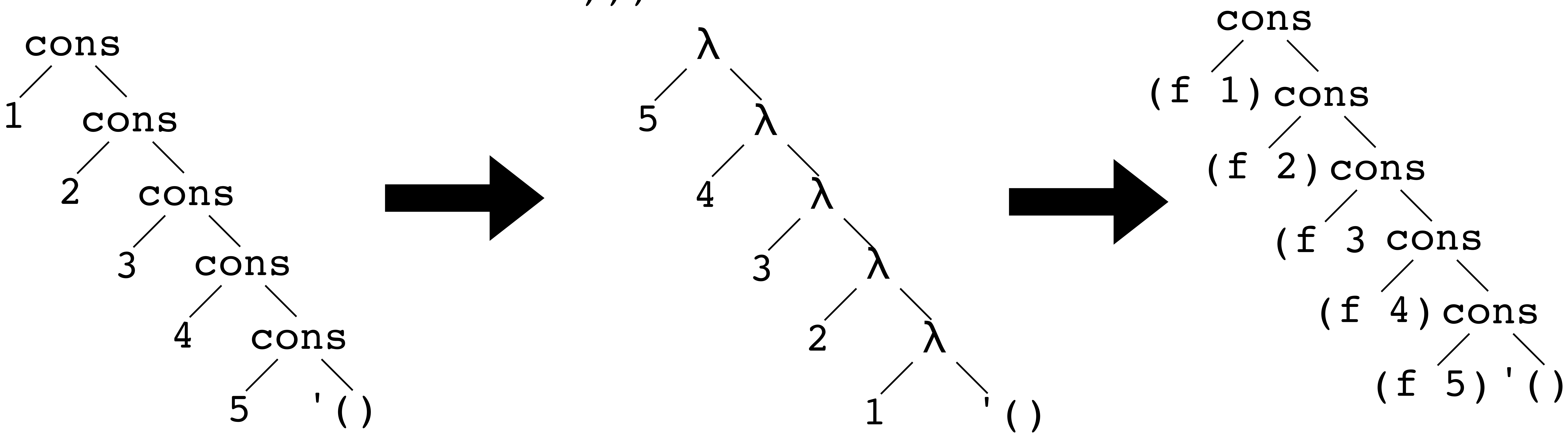
```
(define (reverse lst)
  (foldl cons empty lst))
```



reverse as fold left

`(foldl combine base-case lst)`

```
(define (map f lst)
  (reverse (foldl (λ (head acc)
                    (cons (f head) acc))
                  empty
                  lst)))
```



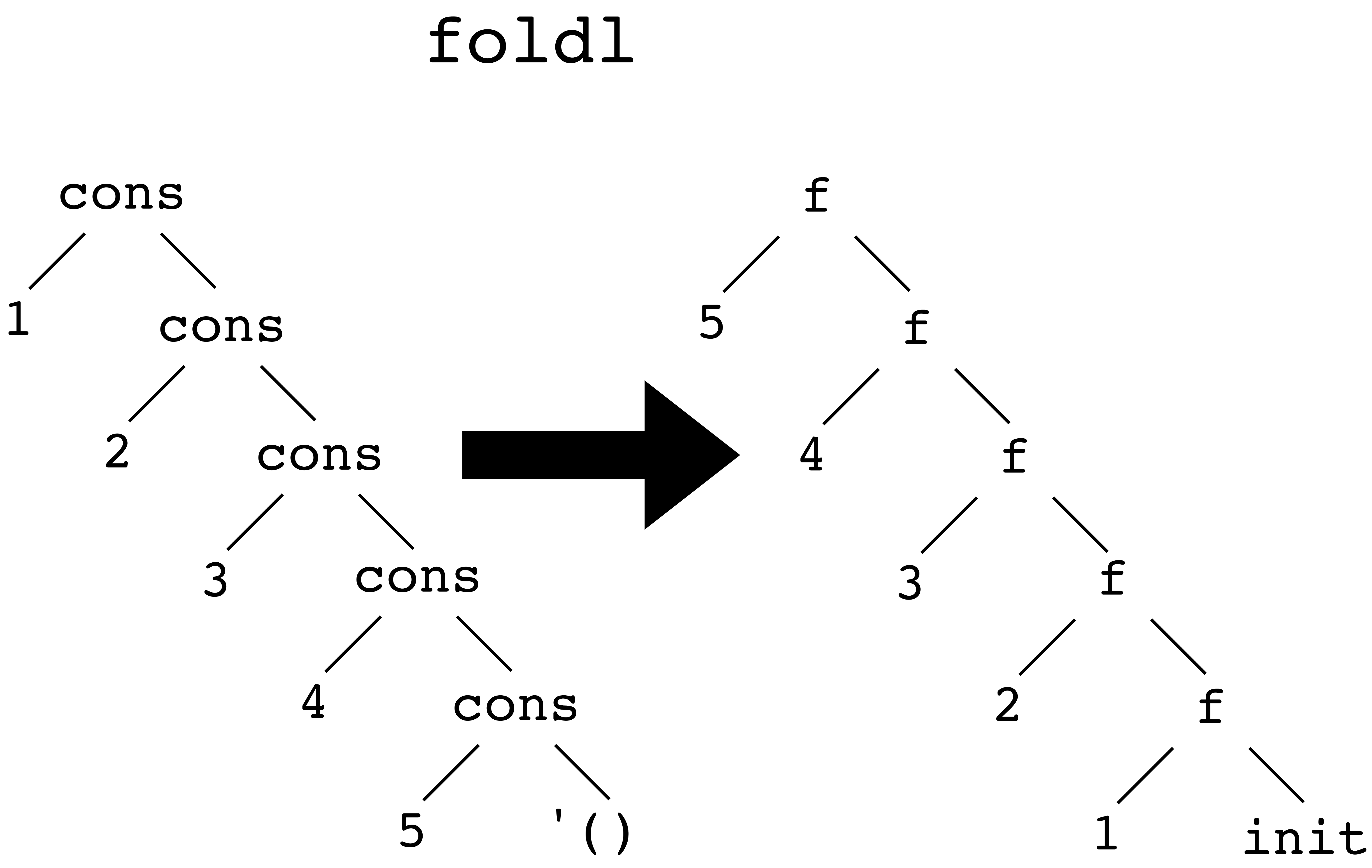
Let's write remove* using `foldl`

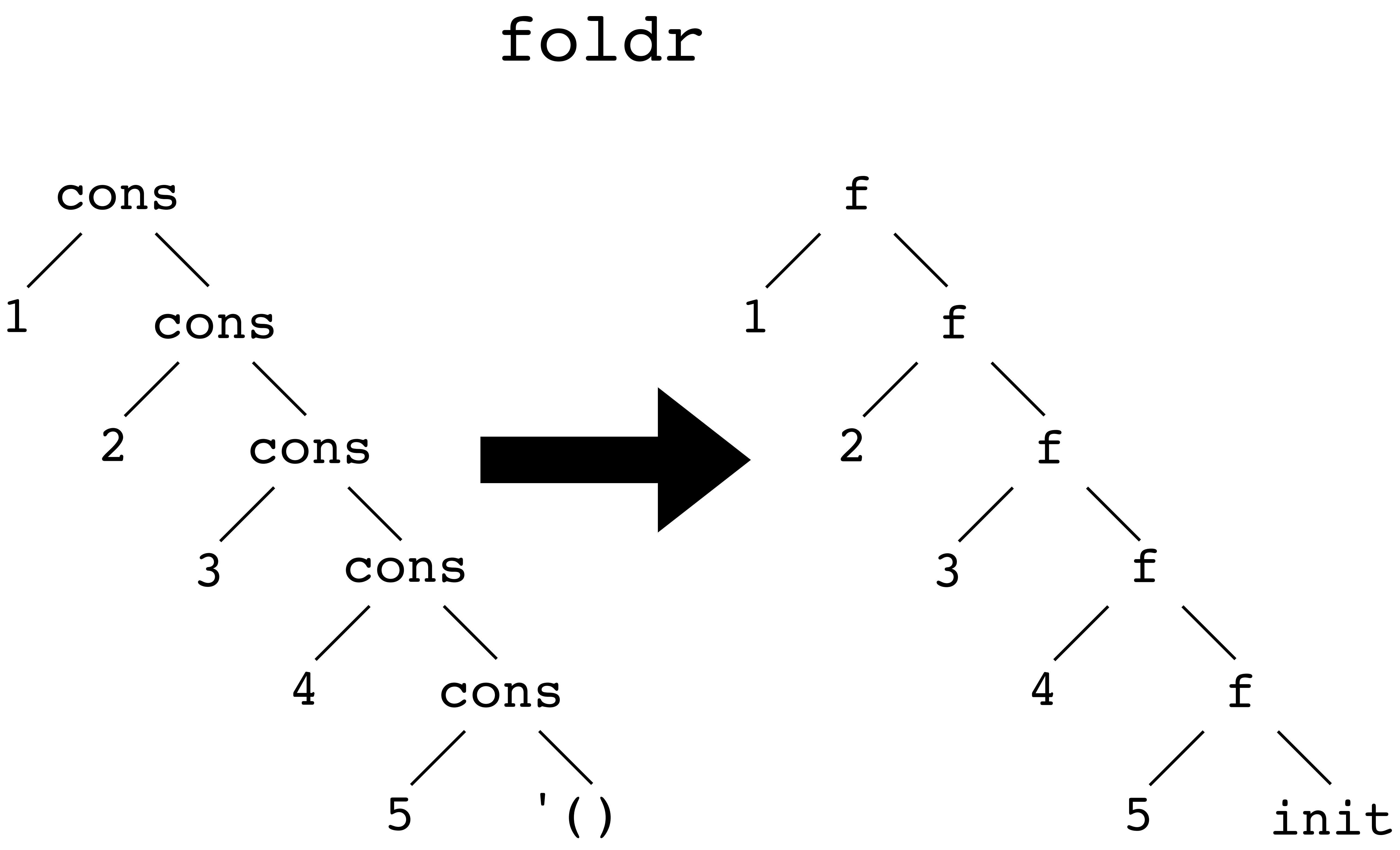
`(foldl combine base-case lst)`

`combine` has the form `(λ (head acc) …)`

We'll need to reverse the result!

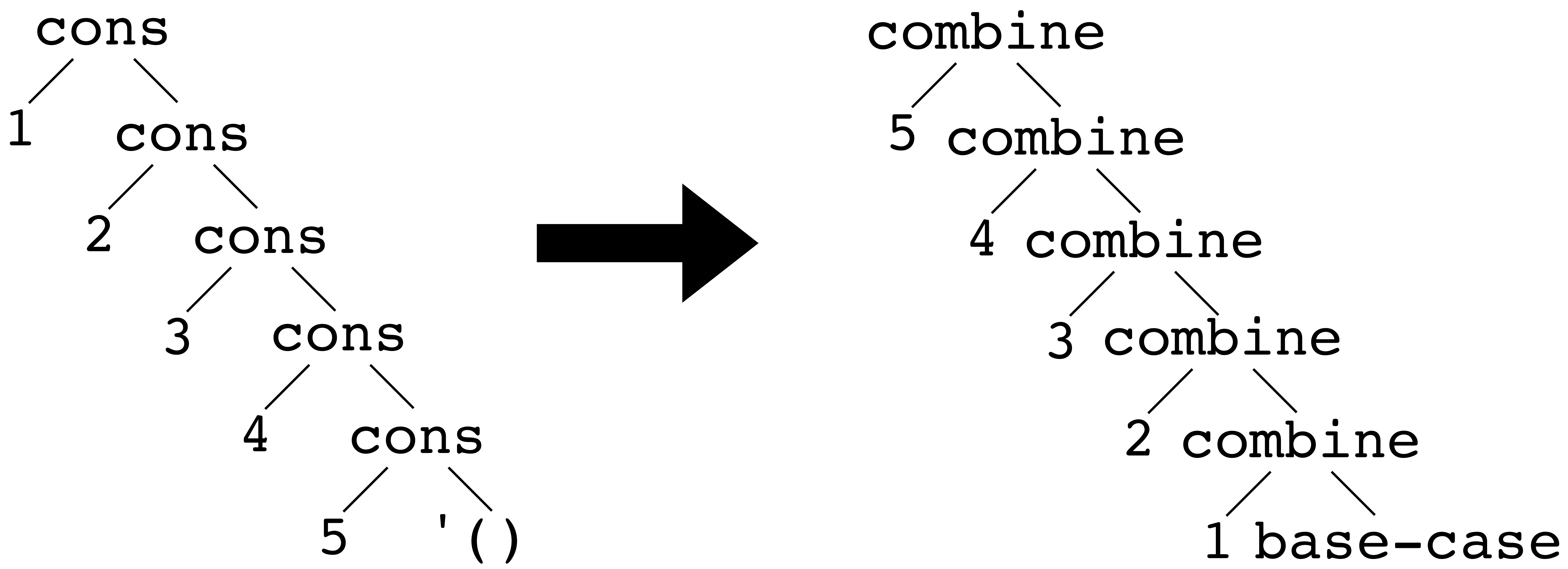
Both folds


foldl
cons
1
cons
2
cons
3
cons
4
cons
5
'()
f
5
f
4
f
3
f
2
f
1
init


foldr
cons
1
cons
2
cons
3
cons
4
cons
5
'()
f
1
f
2
f
3
f
4
f
5
init

Let's write `foldl`

`(foldl combine base-case lst)`



Which is tail-recursive?

```
(define (foldr f init lst)
  (cond [(empty? lst) init]
        [else (combine (first lst)
                       (foldr f init (rest lst)))]))

(define (foldl f init lst)
  (cond [(empty? lst) init]
        [else (foldl f
                     (f (first lst) init)
                     (rest lst))]))
```

A. `foldl`

B. `foldr`

C. Both `foldl` and `foldr`

D. Neither `foldl` nor `foldr`